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Optical Distributed Sensors for Feedback Control: Characterization of Photorefractive Resonator

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Part 1: CHARACTERIZATION OF PHOTOREFRACTIVE RESONATORS

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This report may consist of two parts: Part 1: "Characterization of photorefractive resonators" by G. Indebetouw and Part 2: "Control system design" by D.K. Lindner

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1. INTRODUCTION/AIMS AND GOALS

The aim of the project was to explore, define and assess the possibilities of optical distributed sensing for feedback control. This type of sensors may have some impacts in e.g. the dynamic control of deformable structures (antanae, mirrors) and the monitoring of small displacements (plates, shells).

Conceptually, such a sensor can be divided into three distinct parts: data acquisition, data processing and control design. Analogue optical techniques, because they are noninvasive and afford massive parallelism may play a significant role in the acquisition and the preprocessing of the data for such a sensor. Assessing these possibilities was the aim of the first stage of this project.

The scope of the proposed research was limited to two specific points: (1) the characterization of photorefractive resonators and the assessment of their possible use as a distributed optical processing element (research to be carried out by G. Indebetouw) and (2) the design of a control system utilizing signals from distributed sensors (research to be carried out by D.K. Lindner).

This report summarizes the results of the study of photorefractive resonators. It is presented as follows. A brief summary of the main results of each investigation carried out and their relevance to the aim of the project will be presented. These include a numerical and experimental study of the resonator below threshold (section 2.2), an experimental study of the effect of the resonator's transverse confinement on its dynamics above threshold (sect. 2.3), a numerical study of the resonator above threshold using a modal expansion approach (sect. 2.4) and the experimental test of this model (sect. 2.4.3). A detailed account of each investigation, including methodology and analysis of the results can be found in the four attachments of section 3. (Three reprints of published papers and one preprint of a paper recently submitted for publication). Summary and conclusions are resumed in section 4.

2. PHOTOREFRACTIVE RESONATORS (PCR)

2.1 SYSTEM UNDER STUDY

In order to keep the problem as simple as possible and to extract its most salient features, it was decided to study a PCR with the simplest geometry, i.e.: a linear resonator bounded by an externally pumped photorefractive phase–conjugate mirror (PCM) with gain at one end and a dielectric planar mirror at the other end. This system is described in details in the reprints of sections 3.1, 2 and 3.

For the study below threshold, a plane wave approach was adopted and the resonator had no intracavity elements. Above threshold, the beam's transverse distribution plays a critical role and some intracavity optics and apertures were included to control the resonator's transverse confinement. This kind of resonators, as well as resonators with more complicated geometry (e.g. rings or multiple crystal) may be found useful for temporary image storage, realtime holographic recording, and as part of a recognition or associative system.

2.2 RESONATOR BELOW THRESHOLD

2.2.1 Model

The PCM is an externally pumped photorefractive crystal with a slow response (e.g. $BaTiO_3$). The standard model describing the electro–optic properties of such a crystal was developed by Kukhtarev (Ferroelectrics, 22, 945, 1979). The resonator provides suitable feedback for the oscillating waves and defines the boundary conditions at the crystal entrance face. The fields in the crystal and resonator are assumed to be plane waves. This single–mode approximation is known to be crude but is necessary if analytic solutions are to be derived. Nevertheless, the model did allow for two distinct interaction regions in the crystal, a feature which had not been included in previous analysis of PCRs.

2.2.2 Results

For a material with a slow response, the steady state solution can be extended to define a transfer function of the cavity and analyze its stability. With reasonable approximations, analytical solutions were obtained from which the domain of instability of the resonator could be defined. The transient build–up and decay of the cavity fields were then analyzed, using the full Kukhtarev's model, as a function of the system's parameters (i.e.: PCM gain, cavity and PCM losses, pump ratio and probe ratio). Sets of curves describing the behaviour of the resonator's build–up and decay rates as functions of these parameters were obtained from this analysis. The trends revealed by this numerical analysis were verified experimentally.

2.2.3 Conclusions

The main result of this analysis was to define a procedure by which the PCR can be modeled (at least within the limit of the plane wave approximation) and to describe, a simple means of analyzing the PCR's stability. The trends revealed by the numerical analysis were verified experimentally but only qualitative agreement was found. There are several possible reasons for this: The exact parameter values for the photorefractive crystal used in the experiment were not known; The Kukhtarev's model is based on simplifying assumptions (single carrier type, single photorefractive species) which are known to be only crudely valid for $BaTiO_3$; Gaussian beams were used in the experiments rather than plane waves. In spite of the model's shortcomings, the results of this analysis could be helpful in the initial design of an optical system using PCRs.

2.3 DYNAMICS ABOVE THRESHOLD: TRANSVERSE CONFINEMENT

2.3.1 Transverse profile and optical vortices

If photorefractive resonators have to be used for image storage and processing, the behaviour of the transverse amplitude distribution of the oscillating beam must be understood. Well below threshold, the probe beam distribution determines the resonator's beam profile. Above threshold however, when the PCM gain exceeds the cavity losses, the beam profile depends strongly on the number of transverse degrees of freedom of the cavity (i.e.: on its transverse confinement). An experimental study was carried out to characterize this behaviour.

In order to control the cavity transverse confinement, an afocal geometry with two intracavity pinholes in conjugate planes was used. The set up is described in details in attachments 3.2 and 3.3.

The Fresnel number (F) of the resonator, the square of which is also its space–bandwidth product, is a measure of the maximum number of transverse degrees of freedom of the oscillating beam.

As expected, the transverse complexity of the beam, as measured by the number of lobes or pixels in the beam profile, increases as the square of the Fresnel number. Except for the uniform profile obtained with very small Fresnel numbers, all the patterns observed are dynamic with bright and dark areas executing a dance which can be either periodic (for small F) or chaotic (for larger F). The main frequency of this motion was found to be directly related to the build up rate of the cavity defined and measured in section 2.2.

The most important result of this study was to reveal the presence and motion of optical vortices in the beam. At a vortex, the field amplitude vanishes exactly and around it, the wavefront is helical with a pitch of 2π per turn (vortex charge ± 1). The number of vortices increases as the square of the Fresnel number. They nucleate

spontaneously by pairs of opposite charges in regions of large phase gradient and are never static. They move around in the beam, repelling or attracting each other. They can annihilate one another or disappear at boundaries.

2.3.2 Spatiotemporal dynamics

In order to characterize the dynamics of the PCR as a function of its transverse confinement, local time series of the intensity fluctuations were recorded at one point in the transverse beam profile and were analyzed using power spectra and pseudo phase space portraits. A rich variety of dynamical behaviours was observed. These range from stable output (fixed point) at very small F, to periodic motion (limit cycle), to periodic with an increasing by large number of subharmonics at larger F to aperiodic and eventually chaotic at larger F. At some intermediate value of F, a behaviour reminiscent of intermittancy was also observed. For these experiments, only the Fresnel number was varied by changing the size of one of the two intracavity apertures. All other parameters were fixed.

In order to further characterize one of the chaotic motion (i.e.: that observed at $F = 4.1$), standard algorithms of nonlinear dynamics were applied to the experimental data to calculate the correlation dimension and the entropy. The result, namely $D_2 \sim 5.2$ and $K_2 \sim 0.16\ s^{-1}$, indicates that the chaos observed at this Fresnel number may be deterministic. A spatial correlation index, which crudely measures the spatial coherence of the beam was also measured and shown to drop sharply when the number of vortices in the beam increases.

2.3.3 Conclusions

The results of this experimental study of the PCR allows one to characterize its behaviour as a function of its transverse confinement. Except for very small Fresnel

numbers, this behaviour is found to be complex. Particularly relevant are the dynamic nature of the transverse beam profile and the role played by the vortices. It is clear that this may have far reaching consequences when considering applications where the PCR is used to store images. In order to be able to store a complex image the cavity must have a large space–bandwidth product, thus a large Fresnel number. But at large F, the spatiotemporal behaviour of the beam profile may become chaotic. For such applications, means of locking or stabilizing the transverse profile must be found. Although this possibility cannot be ruled out theoretically, we had no success so far in trying to show that is was practically possible.

An entirely different aspect of this study has revealed the profound analogy that seem to exist between the PCR and other, wildly different, dynamical systems (e.g. fluid flow, physics chemical reactions, lasers,...etc.). This in itself is a fascinating subject and the photorefractive resonator may offer a means of studying the dynamics of other systems or of testing new theoretical conjectures.

2.4. SPATIOTEMPORAL MODEL

2.4.1 Modal expansion approach

In order to be able to make use of a PCR in the design of a sensor, reliable ways of modeling its behaviour must be found. The main difficulty in doing this is the dimensionality of the problem. To adequately represents the role played by the vortices, the two transverse dimensions must be included in the model. It is in principle possible to enlarge the Kukhtarev's model to include these dimensions, but its direct integration would then require a prohibitively large computational budget. Instead, we have tested a modal expansion approach which relies on the assumption that the optical field in the nonlinear crystal can be expanded in a series of the empty cavity eigenmodes and that the number of modes taking part in the dynamics is limited by the cavity Fresnel number. Justifications for this assumption are given in section 3–4.

2.4.2 Numerical results

The modal expansion method was used to predict the Spatiotemporal dynamics of a PCR with modest Fresnel number. As a control parameter for this study we chose to use the off–Bragg mismatch parameter. This parameter measures the momentum mismatch in the four–wave–mixing geometry of the PCM and controls the amount of phase transfer in the four–wave interaction. A bifurcation diagram shows that, as this parameter is varied, the PCR's dynamics changes from periodic, to quasiperiodic with two or more incommensurate frequencies. For some specific parameter ranges, the dynamics can also be chaotic or, in contrast, frequency locking may occur, leading to quieter periodic motions.

One of the main advantage of the modal decomposition approach is that it requires only a modest computational budget to characterize the full spatiotemporal dynamics. As an example, the spatial coherence of the transverse beam intensity fluctuations was studied this way. From this study, an unambiguous correlation between the spatial coherence and the vortices trajectories could be established.

2.4.3 Experimental verification

Experiments were performed to confirm the validity of the modal expansion approach (see section 3–4 for details). A PCR with a Fresnel number close to that used in the numerical analysis was constructed and the experimental off–Bragg parameter was used as a control parameter. A range of behaviours closely resembling the behaviours predicted by the model was observed. These include periodic motions for small off–Bragg parameter, quassiperiodic motions with two or more incommensurate frequencies, aperiodic motions, chaos and frequency locked states.

2.4.4 Conclusions

The modal decomposition approach appears to be a reasonable way of modeling the spatiotemporal dynamics of the PCR. It requires only a modest computational budget and leads to useful physical insights concerning the vortices trajectories and their role in the loss of the beam's spatial coherence as their number increases.

These preliminary results are encouraging but a number of points clearly need further analysis. The most important one is that the choice of the modes which are taking part in the dynamics is somewhat arbitrary in the model. In an experiment, the slightest breaking of symmetry or any small anysotropy may either suppress certain modes or enhance others. Our present state of knowledge of the detailed behaviour of the PCR and the level of our ability to control all its parameters are far too crude for predicting and modeling these factors.

I feel that it is worthwhile to try to determine whether this idea is feasible or not. Thus, for the remaining of the period funded by this grant, I propose to focus on two issues. The first is to document, in full details all the results and findings learned during the course of his project. This work will be the bulk of the PhD dissertation of my student S.R. Liu. The second issue is to determine whether it is possible or not to influence, by external means, the modes that are taking part in the PCR's dynamics. This investigation should be experimental because the available models are too crude. They will be carried out by myself with the help of a starting student who can use this experience as a means of familiarizing himself with experimental methods.

5. List of publications and presentations related to the project

5.1 Papers published in refereed journals

1. "Dynamics of a phase-conjugate resonator: Transient build-up and decay rates" by S.R. Liu and G. Indebetouw, Applied Physics B 54, 247–258 (1992).
2. "Defect-mediated spatial complexity and chaos in a phase-conjugate resonator" by G. Indebetouw and S.R. Liu, Optics Communication 91, 321–330 (1992).
3. "Periodic and chaotic spatiotemporal states in a phase-conjugate resonator using a photorefractive $BaTiO_3$ phase-conjugate mirror" by S.R. Liu and G. Indebetouw, The Journal of the Optical Society of America B9, 1507–1520 (1992).
4. "Spatiotemporal dynamics and optical vortices in a photorefractive phase-conjugate resonator" by S.R. Liu and G. Indebetouw, submitted to the Journal of the Optical Society of America.

5.2 Internal reports (submitted to NASA LRC)

1. "Optical distribution sensors for feedback control–Feasibility study" by G. Indebetouw, October 1990.

2. "Dynamics of a phase–conjugate resonator, Part 1–Transients below threshold" by S.R. Liu and G. Indebetouw, June 1991.

5.3 Presentations

1. "Role of optics in structural control" by G. Indebetouw. Seminar presented at the EE dept., VPI, Oct. 1991.

2. "Dynamics of a phase–conjugate resonator" by G. Indebetouw. colloquium presented at NASA LRC, March 1992.

3. "Spatiotemporal dynamics and optical vortices in a photorefractive phase–conjugate resonator" by S.R. Liu and G. Indebetouw, Poster paper, Optical Society of America Annual meeting, Albuquerque, NM, Sept. 1992.